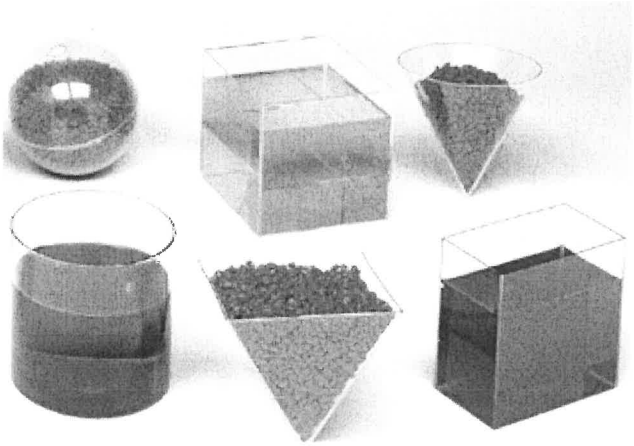
3D GEOMETRY UNIT B:
VOLUME and CAPACITY

Topic	Assignment #	Date
Prisms and cylinders (p. 3-5) **Formulas (p. 4)**	1 (p. 6)	
Pyramids, Cones and Spheres **(p. 9-12)** **Formulas (p. 9-10)**	2 (p. 13)	
Volume and Capacity (p. 17-18)	3 (p. 19)	
Formulas, Unit conversion (p. 20-21)		

Name: _____ _____



Lesson 1: Volume

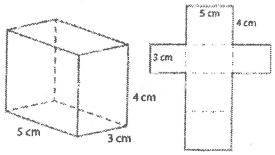
In this lesson, you will:

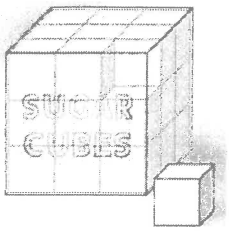
- Calculate volumes using both imperial and metric units
- Convert units of volume from imperial to imperial or metric to metric
- Manipulate a formula to calculate one dimension if the volume is given

Volume - Introduction

Volume and surface area are two different ways to describe 3-D objects.

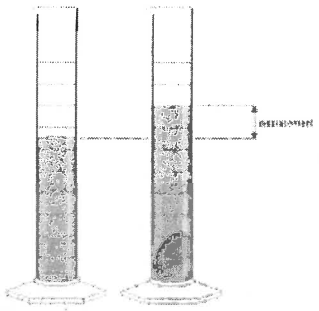
We saw in the previous booklet that **Surface Area** is the AREA of the **OUTSIDE** SURFACES of a 3-dimensional object, measured in **units²**.





Volume is all of the SPACE **INSIDE** a three-dimensional object, or **the amount of space it occupies**, measured in **units³.**

-To find the volume, of a rectangular prism, you could count the number of cubes in it.



-Or if the shape is irregular and not easy to count cubes, you could place it in water. Measure how high the water is before the shape goes in and measure again after.

←The difference is the volume.

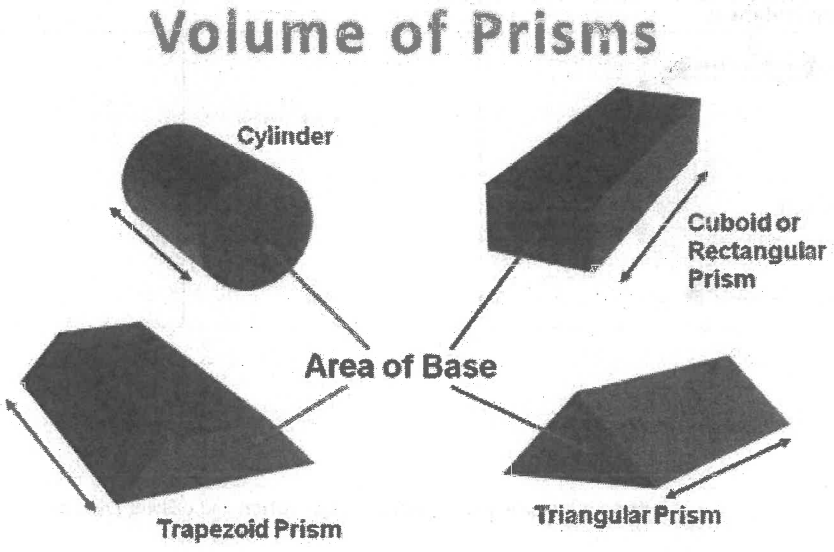
←Cement is delivered in m^3. To know how much cement should be ordered to fill a space, the workers must calculate the **volume** of the space.

Lesson 1: VOLUME (Prisms & Cylinders)

Volume of Prisms

But you can find the volume of any regular prism by **multiplying the area of the base by the height.**

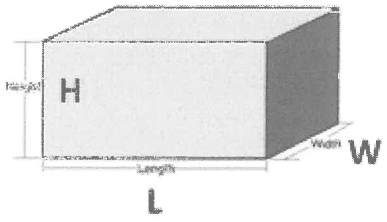
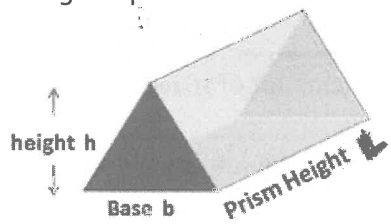
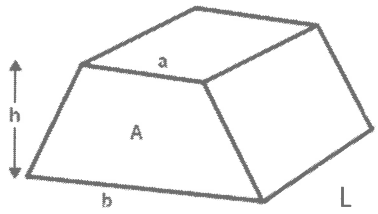
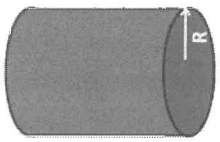
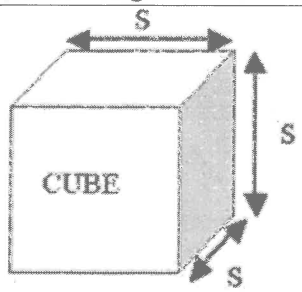
(Remember a **prism** is a 3-D figure where the shape of the base is *consistent* throughout the object. The prism has 2 bases with the same shape. That shape is the **name of the prism.**)



Volume of Prism = Area of Base x Height

3D images Sourced from : http://colinpoyzoue.wordpress.com/

Fill in the chart below – **Volume of Prisms** (*see last pages for formula for area of 2-D shapes*)

prism	Name of base	Formula for area of 2-D base	Volume formula
Rectangular prism 	rectangle	$A = Ll$	$A = Llh$
Triangular prism 	triangle	$A = \frac{bh}{2}$	$A = \left(\frac{bh}{2}\right)L$
Trapezoidal prism  a,b -lengths of parallel lines of trapezoid (marked A) h – height of trapezoid L – length (height) of prism	trapezoid	$A = \frac{(a+b)h}{2}$	$A = \left(\frac{(a+b)h}{2}\right)L$
Cylinder 	circle	$A = \pi r^2$	$A = \pi r^2 H$
	square	$A = s^2$	$A = s^2 H$

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

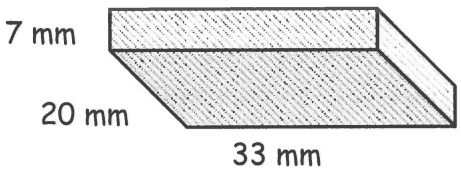
Examples: Volume of Prisms and Cylinders

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

1. Determine the total *volume* (in units³) of the following **prisms**.
(If the answer is infinite decimal, round to 1 decimal place.) (formulas p. 4 and back of booklet)

The volume of a prism is: V = Area of Base x Height

a) Rectangular Prism (4620 mm³)

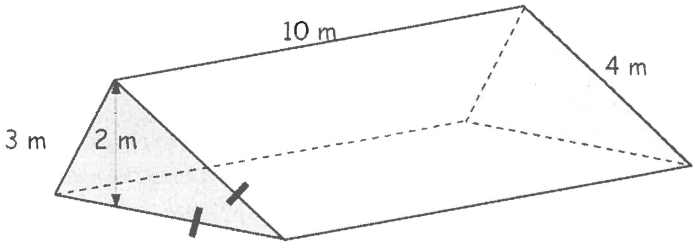


$V = LWH$

$= (33)(20)(7)$

$V = 4620\ mm^3$

b) Triangular Prism (40m³)



$V = \left(\frac{bh}{2}\right)L$

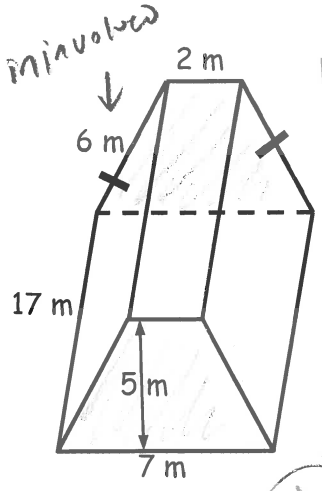
$= \left(\frac{4 \times 2}{2}\right)10$

$= (8 \div 2)10$

$= 4(10)$

$V = 40\ m^3$

c) Trapezoidal Prism (382.5 m³)



$V = \left[\left(\frac{a+b}{2}\right)h\right]L$

$= \left(\frac{2+7}{2}\right)(5)(17)$

$= (9 \div 2)(5)(17)$

$= (4.5)(5)(17)$

$V = 382.5\ m^3$

(538.8 cm³)

2. Determine how much cola could fit in the following container.
(Hint: find the *volume* of the following **cylinder**)

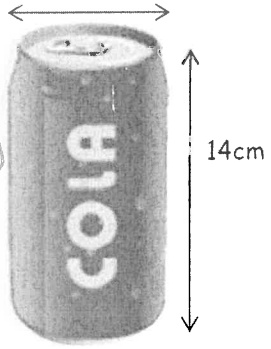
$d = 7$

$r = 3.5$

$V = \pi r^2 h$

$= \pi(3.5)^2(14)$

$V = 538.8\ cm^3$

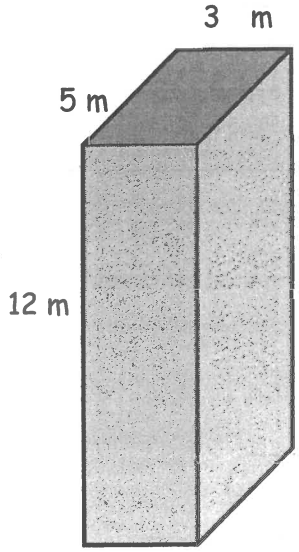


Assignment 1: VOLUME (Prisms & Cylinders)

* Show all your work (equation, substitution and answer) and place a box around your final answer

1. Determine the total *volume* the following **prisms**:

a)



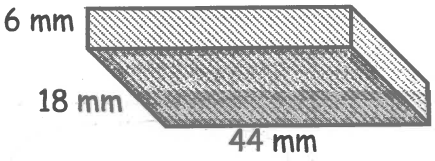
L = 5m W = 3m H = 12m

V = Lwh

V = 5 x 3 x 12

V = 180 m^3

b)



L = 44m W = 18mm H = 6mm

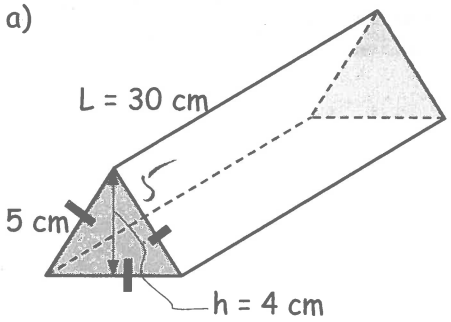
V = Lwh

V = 44 x 18 x 6

V = 4752 mm^3

2. Determine the total volume of the following **prisms (triangular)**:

a)



V = Area of base (triangle) x Height of prism

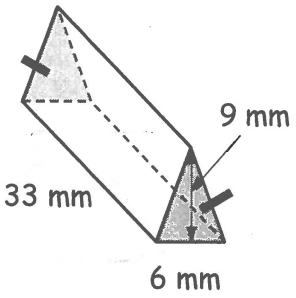
V = $\left(\frac{bh}{2}\right)L$

b = 5 h = 4 L = 30

V = [(5 x 4) ÷ 2] [30]

V = 300 cm³

b)



V = $\left(\frac{bh}{2}\right)L$

b = 6 h = 9 H = 33

V = [(6 x 9) ÷ 2] x 33

V = 891 mm³

3. Determine the total *volume* of the following **prisms (cylinders)**: (round to 1 decimal)

a)

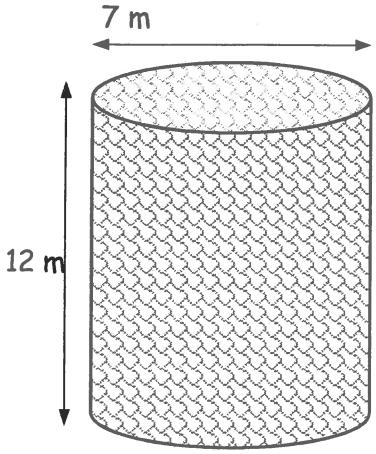
V = Area of base (circle) x Height of cylinder

V = $\pi r^2 h$

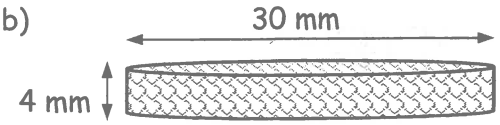
r = 3.5 h = 12

V = π x (3.5^2) x 12

V = 461.8 m³



b)



$V = \pi r^2 h$

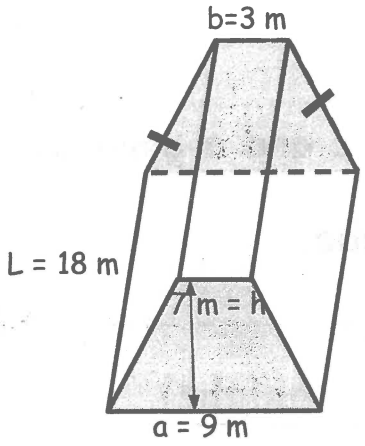
$r = 15$ $h = 4$

$V = \pi \times (15)^2 \times 4$

$V = 2827.4 \text{ mm}^3$

4. Determine the total volume of these **trapezoidal** prisms.

$a = 9$ $b = 3$ $h = 7$ $L = 18$



V = Area of base (trapezoid) x Height of prism

$V = \left[\left(\frac{a+b}{2}\right) h\right] L$

$V = \left[\left(\frac{9+3}{2}\right) 7\right](18)$

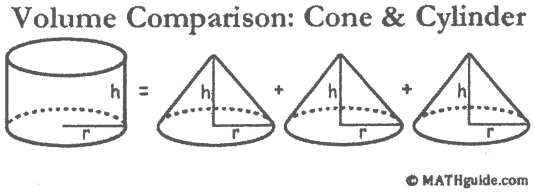
$V = 756 \text{ m}^3$

Answers:

1) 180 m^3, 4752 mm^3,
2) 300 cm^3, 891 mm^3
3) 461.8 m^3, 2827.4 mm^3
4) 756 m^3

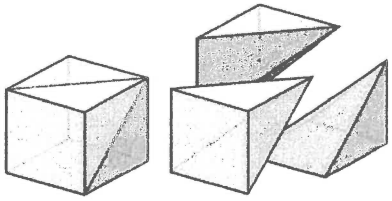
Lesson 2: VOLUME (Pyramids, Cones and Spheres)

Volume of Pyramids and Cones

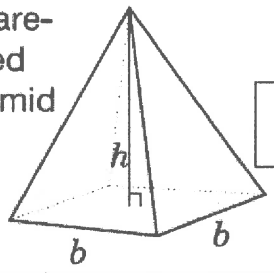


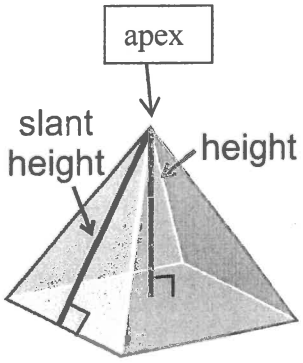
You can put 3 cones together to form a cylinder.

Similarly, you can put 3 square-based pyramids together to form a cube.



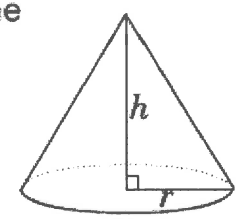
For that reason, the formulas for volume of a cone and a pyramid are 1/3 times the formula for cylinder and for cube.

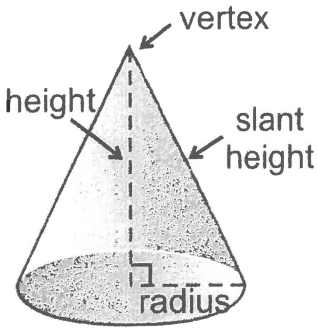
Square-based pyramid	
	$V = \frac{(\text{area of base})(\text{height})}{3}$ Shape of base: square $V = \frac{1}{3}b^2h$ or $V = \frac{b^2h}{3}$



Note:

- For **surface area** of pyramid , we use the **slant height (s)** (*height of the triangle*).
- For **volume** of a pyramid, we use the **vertical height (h)** of the pyramid from the base to the apex *(if we were inside the pyramid, the height is from the floor straight up to the top point)*

Cone	
	$V = \frac{(\text{area of base})(\text{height})}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$



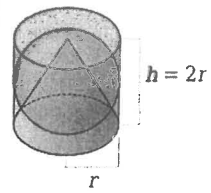
Similar to the height (h) and slant height (s) of pyramid,

- the **height of the cone (h)** is the **vertical height** from the base of the cone to the vertex (top point) of the cone. The height is perpendicular to the radius (*the height and the radius form a 90° angle*).
- the **slant height (s)** is the distance from the circle base to the vertex

Volume of Sphere

(In case you were wondering about the formula for the volume of a sphere...

The volume of a cone and sphere fit together make a cylinder (assuming they are made to perfectly fit each other, so the height is twice the radius). The cone is 1/3 the volume of the cylinder and so the **sphere is 2/3 the volume of the cylinder**. If you make height = 2 x radius, you can come up with the formula for the volume of a sphere.



Volume = 2/3 * volume of cylinder

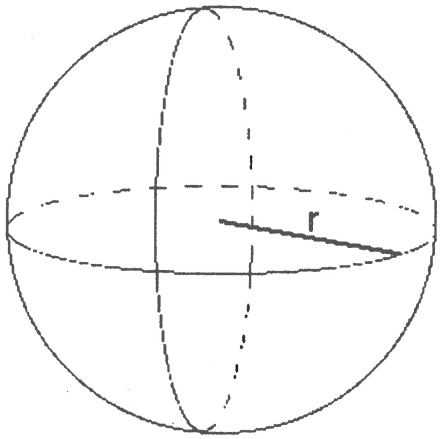
= 2/3 * $\pi(radius)^2 * height$

= 2/3 * $\pi(radius)^2 * (2 * radius)$

$= \frac{4}{3}\pi(radius)^3$

Therefore the formula for the

Volume of a Sphere is:



$$\text{Volume} = \frac{4}{3}\pi r^3$$

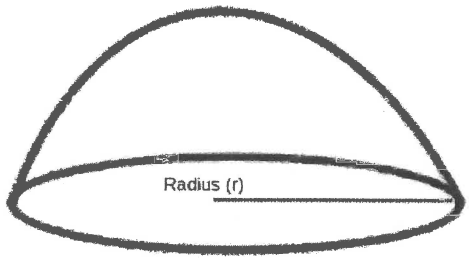
And then the volume of a hemisphere *(half a sphere)* is simply ½ times the volume of a sphere.

What is the VOLUME of a Hemisphere?

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2\pi}{3}R^3$$

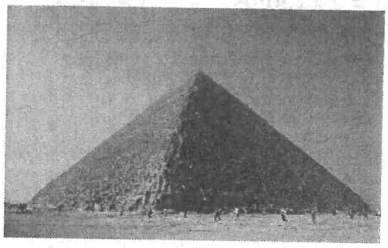


Hemisphere Mathgun.com

Examples VOLUME - Pyramids, Cones and Spheres

* Show all your **work** (equation, substitution and answer) and place a **box** around your final answer *

1. One of the famous pyramids built by the Egyptians is the great pyramid of Giza. Determine how much sand was used to build the pyramid (i.e. determine the total *volume* of the **pyramid**)



Height = 146m h

Square base length = 230m b

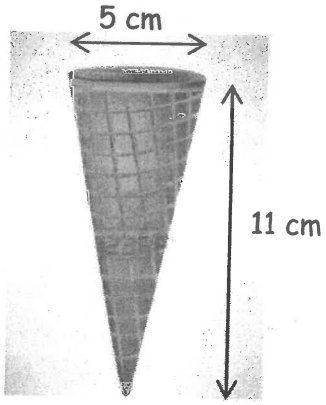
$$V = \frac{b^2 h}{3}$$

$$= [(230)^2 (146)] \div 3$$

$V = 2\,574\,466.7\ m^3$

2. Determine how much ice cream could fit into this cone before you put on the top scoop(s) (i.e. determine the total volume of the **cone**) round to 1 decimal.

$r = 2.5$

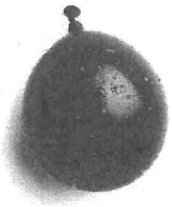


$$V = \frac{\pi r^2 h}{3}$$

$$= [\pi (2.5)(11)] \div 3$$

$V = 28.8\ cm^3$

3. Assume this water balloon is a perfect sphere. Determine the volume of water that is inside the balloon if its radius is 7cm. round to 1 decimal



$$V = \frac{4}{3}\pi r^3$$

$$= [4\pi (7)^3] \div 3$$

$V = 1436.8\ cm^3$

use exponent button on calculator
y^x
x^y
$\wedge$

4. Find the volume of ice in this "snow cone" if the cone itself is also full.

The radius is 80 mm and the height of the cone is 90 mm.

V = half sphere + cone

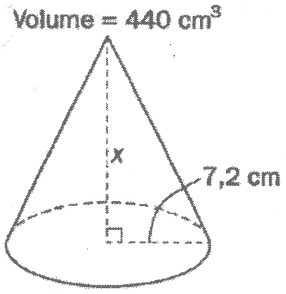
$= \left(\frac{4}{3}\pi r^3\right) \div 2 + \frac{\pi r^2 h}{3}$

$= \frac{[4\pi(80)^3]}{3} \div 2 + \frac{\pi(80)^2(90)}{3}$

$= 1072330.3 + 603185.8$

$V = 1675516.1\ mm^3$

5. Write out the formula for the volume of a cone and then substitute in the given values for volume and radius. Use algebra to solve to find the height of the cone.



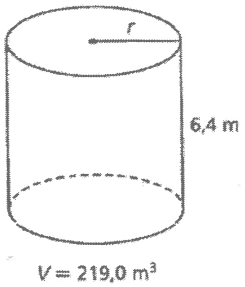
$V = \frac{\pi r^2 h}{3}$

$440 = \frac{\pi(7.2)^2(h)}{3}$

$\frac{440}{54.3} = \frac{54.3h}{54.3}$

$h = 8.1\ cm$

6. Use a similar method as #5 to find the radius of this cylinder.



$V = \pi r^2 h$

$219 = \pi r^2 (6.4)$

$\frac{219}{20.1} = \frac{20.1r^2}{20.1}$

$\sqrt{r^2} = \sqrt{10.9}$

square root of both sides because $\sqrt{r^2} = r$

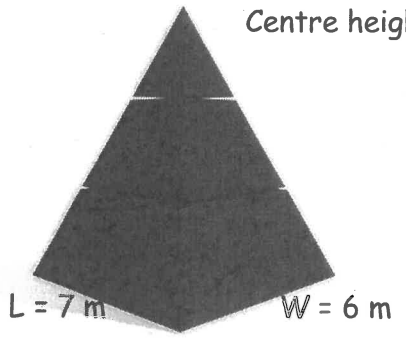
$r = 3.3\ m$

Assignment 2: VOLUME (Pyramids, Cones and Spheres)

* Show all your work (equation, substitution and answer) and place a box around your final answer *

1. Determine the total *volume* of the following **pyramids**:

a) Centre height (H) = 8 m



V = 1/3 Area of base (rectangle) X Height of pyramid

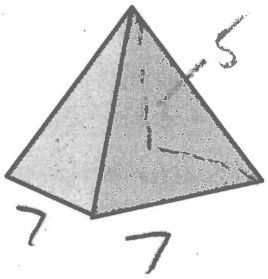
L = 7 W = 6 H = 8

$V = \frac{1}{3} L W h$

$V = \frac{(7)(6)(8)}{3}$

V = 112 m^3

b) Square base length = 7 in, height = 5 in



L = 7 W = 7 H = 5

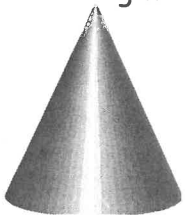
$V = \frac{1}{3} L W H$

$V = \frac{1}{3} (7)(7)(5)$

V = 81.7 in^3

2. Determine the total volume of the following **cones**:

a) centre height = 35 cm



r = 10

V = 1/3 Area of base (circle) X Height of cone

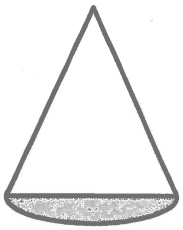
$V = \frac{1}{3} \pi r^2 h$

r = 10 H = 35

$V = \frac{\pi (10)^2 (35)}{3}$

V = 3665.2 cm^3

b) Height = 70 yards, radius = 50 yds



$V = \frac{1}{3}\pi r^2 h$

$r = 50$ $H = 70$

$V = \frac{\pi(50)^2(70)}{3}$

$V = 183\,259.8\ yd^3$

3. Determine the volume of the following **spheres**.

a) Radius = 1.5 dm

$V = \frac{4}{3}\pi r^3$ $r = 1.5$

$V = \frac{4\pi(1.5)^3}{3}$

$V = 14.1\ dm^3$

b) The diameter of the ping-pong ball is 3.5 cm.

$V = \frac{4}{3}\pi r^3$ $r = 1.75$

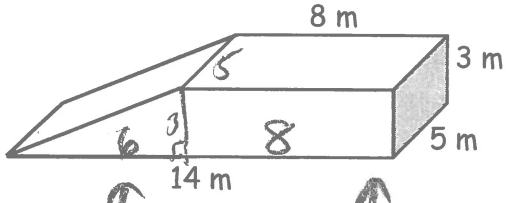
$V = \frac{4}{3}\pi(1.75)^3$

$V = 22.4\ cm^3$

CHALLENGE!

4. Determine the volume of these *composite* objects:

a)

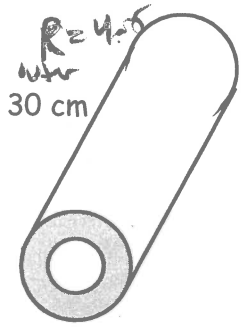


$b = 6$, $h = 3$, $L = 5$

$L = 8$, $W = 5$, $h = 3$

$$V = \left(\frac{bh}{2}\right)L + LWH$$
$$= \frac{(6)(3)}{2}(5) + (8)(5)(3)$$
$$= 45 + 120$$
$$V = 165\ m^3$$

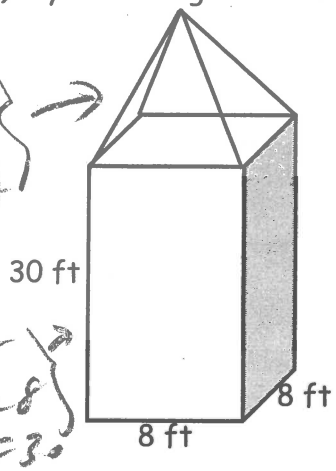
b) $D_{out} = 9$ cm, $D_{in} = 5$ cm



$R = 4.5$ outer, $r = 2.5$ inner

$V =$ big cylinder − inner cylinder
$$= \pi R^2 h - \pi r^2 h$$
$$= \pi(4.5)^2(30) - \pi(2.5)^2(30)$$
$$= 1908.5 - 589.0$$
$$V = 1319.5\ cm^3$$

c) Pyramid height at the centre = 7 ft



$L = 8$, $W = 8$, $h = 7$

$L = 8$, $W = 8$, $H = 30$

$$V = \frac{1}{3}LWH + LWH$$
$$= \frac{1}{3}(8)(8)(7) + (8)(8)(30)$$
$$= 149.3 + 1920$$
$$V = 2069.3\ ft^3$$

Answers:

1) 112 m^3, 81.7 in^3
2) 3665 cm^3, 183 ~~260~~ 59.6 yd^3
3) 14.1 dm^3, 22.4 cm^3
4) 165 m^3, ~~43.98~~ 1319.5 cm^3, 2069.3 ft^3

5. The volume of a sphere is 41 m³. What is the diameter of the sphere?

$V = \frac{4}{3}\pi r^3$

$41 = \frac{4}{3}\pi r^3$

$\frac{41}{4.1} = \frac{4.1\,r^3}{4.1}$

$\sqrt[3]{10} = \sqrt[3]{r^3}$

$2.2 = r$

$r = 2.2$

diameter = 4.4 m

cube root both sides $\sqrt[x]{y}$ button

Push $\sqrt[x]{y}$ then 3.

6. The volume of a rectangular prism is 720 mm³. If the width is 6 mm and the height is 10 mm, what is the length?

$V = LWH$

$720 = L(6)(10)$

$\frac{720}{60} = \frac{60L}{60}$

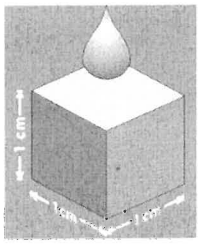
$12 = L$

The length is 12 mm.

Volume and Capacity

We have seen that the **volume** measures the **amount of space** inside an object.

Capacity is the amount of *liquid* that an object can contain ***(the amount a container can hold).*** Capacity is usually measured in **millilitres (ml)** or **litres (l)** **(Common imperial units are ounces, gallons, pints, and in cooking -cups, tbsp, tsp).** The amount of ml in a can of soda or litres of milk is the **capacity of liquid** in that container – how much liquid it can hold. In everyday life we refer to capacity when cooking (how many cups, ounces), when buying food **(how many ml is in the can)**, when buying gas (in litres), etc.



*A cylindrical can of pop has a **volume** of **375 cm³**, but is labeled as **375 ml** because it contains **liquid**.*

1 ml of liquid fits exactly in a cube 1cm x 1cm x1cm **(1cm³)**

1 cm³ = 1 millilitre
1000 ml = 1 litre.
1 000 cm ³ = 1 litre
1m³ = 1 kilolitre

To find capacity,

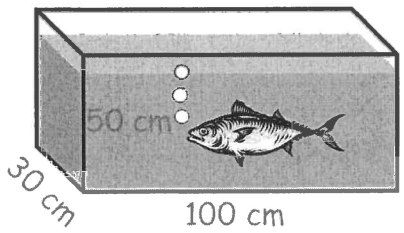
1. Find the volume in cm³.
2. Convert your cm³ answer to ml (same numbers; different unit).

- To find how many litres equal volume in cm³,
  - First convert cm³ to ml, (=) then convert ml to L (x1000).
    or:
  - Find the answer in m³, convert to kL (=), then convert kL to L (÷1000).

- To find out how many ml or litres in an object with dimensions in metres,
  - **1. C**onvert the dimensions to cm.
  - 2. Follow the steps above.

Example

How much water can this fish tank hold in litres?



1cm³ = 1 mL
1000 ml = 1 litre
1 m³ = 1 kL

L=100÷100
=1m

Method 1 (find cm³, convert to ml, convert to L)

L=100cm W=30cm H=50cm
V=LWH
=(100)(30)(50)
V= 150 000 cm³
V= 150 000 ml
V= 150 000 ÷1000 L
V= 150 L

Method 2 (convert dimensions to m, find m³, convert to kL, convert to L)

V=LWH
=(1)(0.3)(0.5)
V= 0.15 m³
V= 0.15 kl
V= (0.15)(1000) L
V= 150 L

W= 30÷100
=0.3m
L= 50÷100
=0.5m

Try it!

1 A can of soup has a volume of 460 cm^3. What the capacity of the can in ml? 460 ml
What is the capacity in litres? 0.46 L

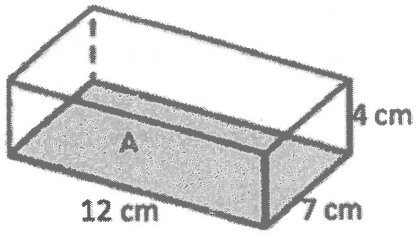
460 cm^3 = 460 ml

460 ml ÷ 1000 = 0,46 L

2 A plastic box measures 24 cm wide, 33 cm long, and 12 cm high. What is the capacity of the box, in litres?

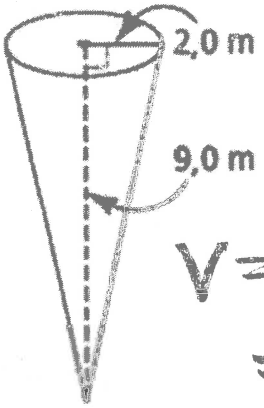
V = LWH
= (24)(33)(12)
= 9504 cm^3
= 9504 ml
9504 ÷ 1000 = 9.504
V = 9.504 L

3 What is the capacity of the reservoir, in ml?



V = LWH
= (12)(7)(4)
V = 336 cm^3
V = 336 ml

4 What is the capacity of the cone-shaped container, in litres?

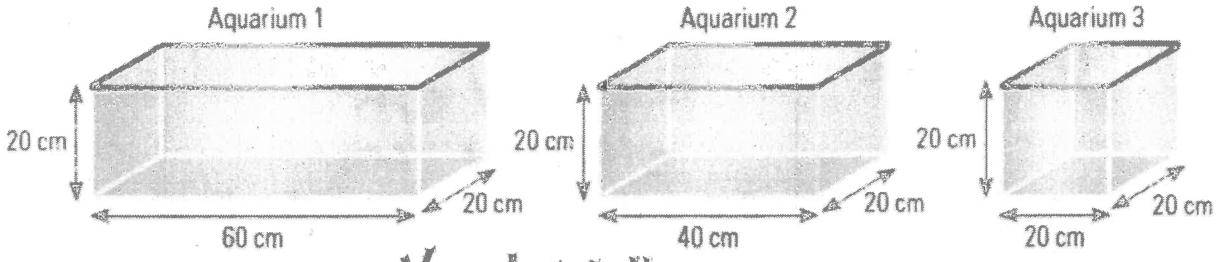


$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(2)^2(9)$
V = 37.7 m^3
V = 37,7 kl
37,7 × 1000 = 37 700
V = 37 700 l.

Assignment 3: VOLUME and CAPACITY

* Show all your work (equation, substitution and answer) and place a box around your final answer

1. Aaliyah has 3 aquariums with the following dimensions. She needs to buy distilled water to fill the aquariums.



$V = LWH$

a) Calculate the total volume of the 3 aquariums, in cm^3.

$V = 20 \cdot 60 \cdot 20$ $V = 24\,000\ cm^3$

$V = 40 \cdot 20 \cdot 20$ $V = 16\,000\ cm^3$

$V = 20 \cdot 20 \cdot 20$ $V = 8000\ cm^3$

b) What is the total capacity of the 3 aquariums together, in litres?

$V = 24\,000 + 16\,000 + 8000$

$V = 48\,000\ cm^3$

$V = 48\,000\ ml$

$V = 48\,000 \div 1000\ L$

$V = 48\ L$

The 3 aquariums have a capacity of 48 L.

c) Aaliyah can buy ~~4 litre bottles~~ of distilled water for $3.95 and 2 litre bottles of distilled water for $2.19, taxes included. Aaliyah wants to ~~buy~~ pay the least ~~water~~ possible. How much does she have to pay to fill the aquariums with water?

$48\ l \div 4 = 12$ four L bottles

$12 (3.95)$

$= \$47.40$

$48 \div 2 = 24$ 24 two L bottles

$24 (2.19)$

$= \$52.56$

She has to pay $47.40.

Area of 2D objects: in units²

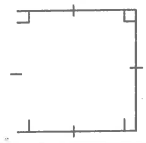
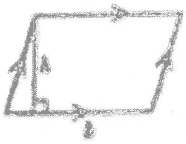
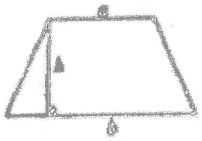
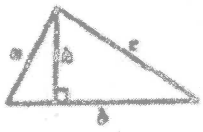
Name of Formula	Diagram	Formula
area of a square		$A = s^2$
area of a rectangle		$A = lw$
area of a parallelogram		$A = bh$
area of a trapezoid		$A = \frac{(a+b)h}{2}$ / $A = \frac{1}{2}(a+b)h$
area of a triangle		$A = \frac{bh}{2}$ / $A = \frac{1}{2}bh$
area of a circle		$A = \pi r^2$ (π button on calculator)

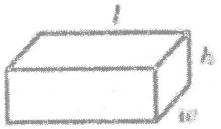
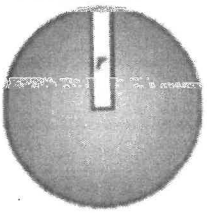
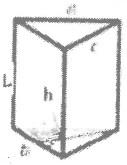
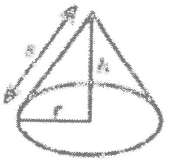
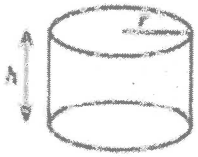
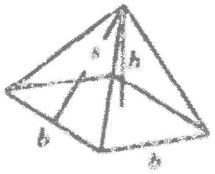
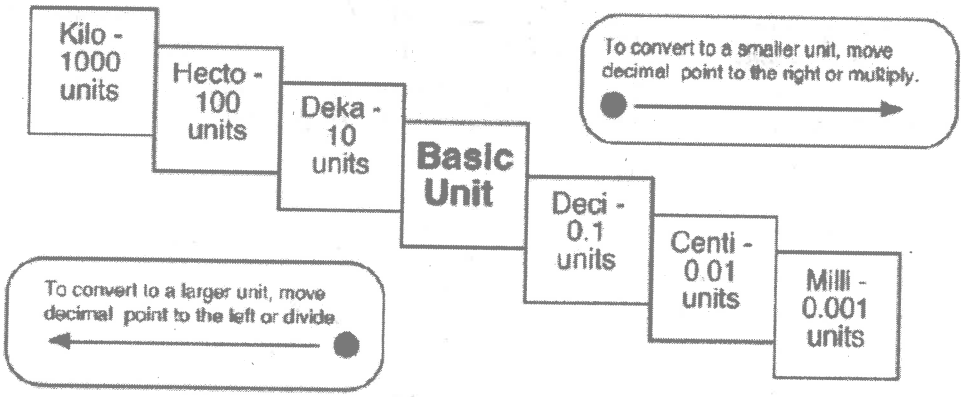
Figure	Diagram	Surface Area (in square units)	Volume (in cubic units)
rectangular prism		$SA = 2wh + 2lw + 2lh$ (sides; top/bottom; front/back)	$V = lwh$
sphere		$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Figure	Diagram	Surface Area (in square units)	Volume (in cubic units)
Triangular Prism		$SA = 2(\frac{bh}{2}) = aL + bL + cL$ bases sides (isoc. Δ - 2 sides same area)	$V = (\frac{bh}{2})L$
cone		$SA = \pi rs + \pi r^2$ **(slanted side only)**	$V = \frac{1}{3}\pi r^2 h$
cylinder		$SA = 2\pi rh + 2\pi r^2$ Side bases	$V = \pi r^2 h$
square base pyramid		$SA = b^2 + 2sb$ base sides (s - slant height h – vertical height from apex to base of pyramid	$V = \frac{1}{3}b^2 h$

Metric Conversion Chart



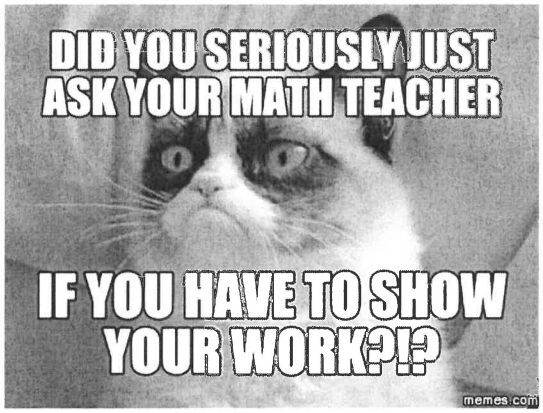
1 km = 1000 m
1 kg = 1000 g

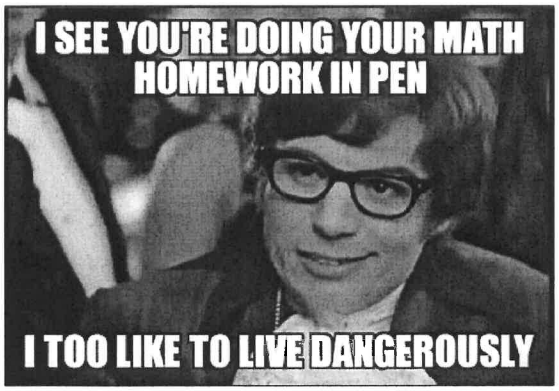
1 m = 100 cm
1 cm = 10 mm

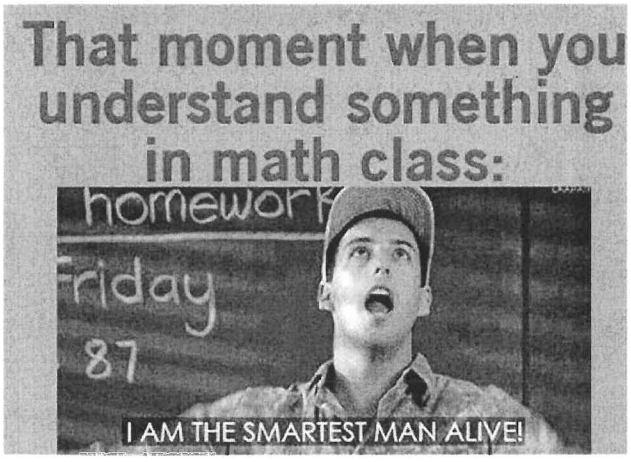
1 g = 1000 mg
1 ℓ = 1000 ml

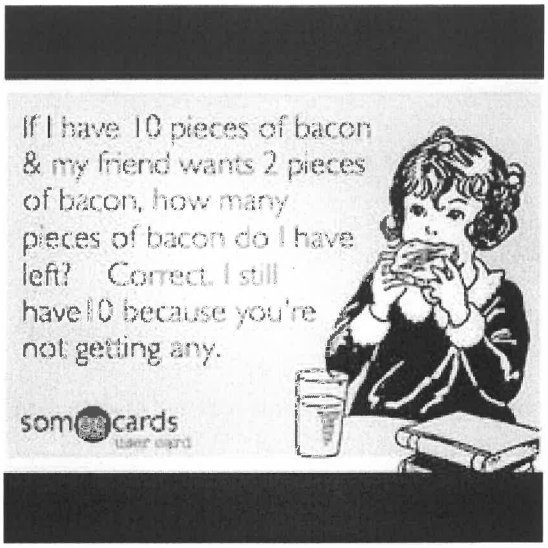
Math = Love

THINGS TEENAGERS SAY

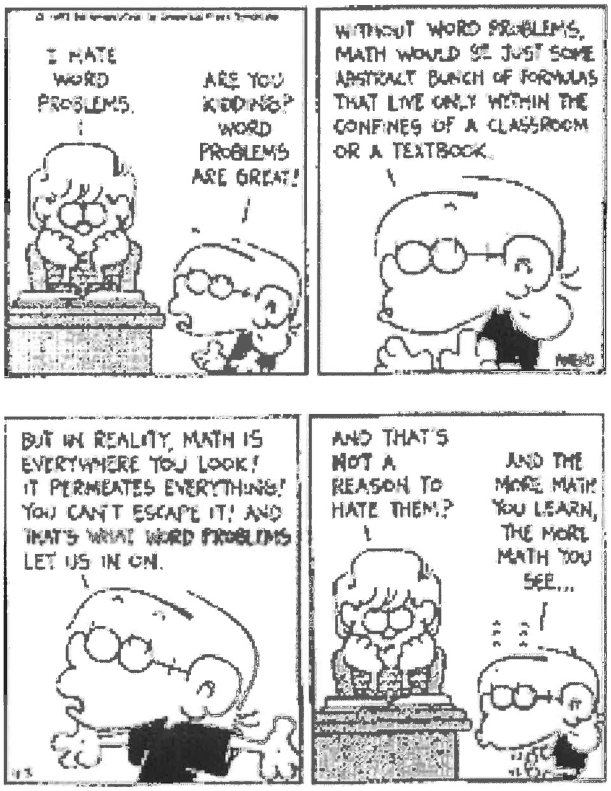








Fox Trot



MISTAKES *are proof that you are* TRYING